ELTE DOCTORAL SCHOOL OF INFORMATICS
COMPLEX EXAM TOPICS

Approximation theory (recommended subject)

1. Fundamental theorems of Approximation Theory: Weierstrass' First and Second Approximation Theorem. The reciprocal coherence between the two Weierstrass Theorems.

2. The basic problems of the approximation theory in normed linear spaces. Approximation in Banach and Hilbert spaces: existence, uniqueness and the determination of the best approximation. The minimum property of the Fourier projection in Hilbert spaces.

3. Function spaces. Function spaces of continuous functions. Function spaces of integrable functions. Weighted function spaces. Existence and uniqueness of the best approximation in these spaces.

4. Chebyshev approximation: Chebyshev alternation theorems. Deriving of Chebyshev polynomials and their fundamental properties.

5. Modulus of continuity and smoothness in he function spaces $C_{2\pi}$ and $L^{p}{}_{2\pi}$. Direct or Jackson type theorems on the approximation of periodic functions. Inverse or Bernstein type theorems on the approximation of periodic functions.

6. Approximation by algebraic polynomial. The order of the approximation by Bernstein polynomials. The problem of the saturation. The Ditzian-Totik modulus of smoothness. Direct and inverse theorems.

7. The approximation properties of the trigonometric Fourier partial sums. Summation processes of the trigonometric Fourier series. The Natanson-Zuk theorem for the uniform convergence. Classical uniformly convergent processes. The order of the convergence for the summations of trigonometric Fourier series.

8. Constructions of uniformly convergent algebraic polynomial sequences. Orthogonal polynomilas. Classical orthogonal polynomilas. Fourier series with respect to orthogonal polynomials and their summations.

9. Interpolation with algebraic and trigonometric polynomials. Lagrange, Hermite and Hermite-Fejér interpolation. The convergence of the interpolation processes. The role of the Lebesgue function. Weighted interpolation. Summation of discrete linear processes and their convergence.

References

1. E. W. Cheney: *Introduction to Approximation Theory*, McGraw-Hill, New York, (1966).
2. E. W. Cheney and W. Light: *A Course in Approximation Theory,* Brooks/Cole Publ. Comp., 2000.
3. R. A. DeVore: *The Approximation of Continuous Functions by Positive Linear Operators,* Lecture Notes in Mathematics, 293, Springer-Verlag, Berlin, 1972.
4. R. A. DeVore and G.G. Lorentz: *Constructive Approximation*, Springer-Verlag, New York, 1993.
5. G.G. Lorentz, M. von Golitschek and Y. Makovoz: *Constructive Approximation: Advanced Problems,* Springer-Verlag, New York, 1996.
6. I.P. Natanson, *Constructive Function Theory*, I. ,II., III., Frederick Ungar Publishing Co., New York, 1964.
7. P. P. Petrushev and V. A. Popov: *Rational Approximation of Real Functions,* Cambridge Univ. Press, New York, 1987.
8. A. F. Timan: *Theory of Approximation of Functions of a Real Variable,* Macmillman, New York, 1963. Reprint, Dover, New York.
9. V.K. Dzyadyk and I.A. Shevchuk, *Theory of Uniform Approximation of Functions by Polynomials*, Walter de Gruyter, Berlin - New York, 2008.