DOCTORAL SCHOOL OF INFORMATICS
COMPLEX EXAM SUBJECT

Complexity theory (main subject)

The Turing machine. The definition and existence of the universal Turing machine. A k-tape Turing machine can be simulated by a single-tape Turing machine in $O(N^2)$time. The RAM machine. Recursive and recursively enumerable languages. Time- and space-bounded language classes. DSPACE(f(n)), DTIME(f(n)), P, PSPACE. The linear speed-up theorem. For every recursive function f, there exists a recursive language that is not in DTIME(f(n)) (that is, arbitrarily hard languages exist).

The nondeterministic Turing machine. Nondeterministic language classes.
Definition of NP-completeness. Cook's theorem: SAT is NP-complete.

Randomized algorithms, randomized complexity classes, randomized Turing machines. A randomized algorithm for deciding the existence of a perfect matching in a bipartite graph. Primality testing: naive methods, the Fermat test, the Miller–Rabin test. The Agrawal–Kayal–Saxena test: primes are in P (without proof).

LOGSPACE, NLOGSPACE. REACHABILITY is in NLOGSPACE, PALINDROMES are in LOGSPACE. REACHABILITY is in DSPACE($\log^2 n$). Savitch's theorem. PSPACE-completeness, the TQBF language.

Interactive games and proofs (Babai, Micali, Rackoff). The class IP. Shamir's theorem: IP = PSPACE. Communication games, the Mehlhorn–Schmidt theorem. Nondeterministic communication complexity. Randomized communication games. The Aho–Ullman–Yannakakis theorem.

Boolean circuits: size, depth, fan-in. Uniform and non-uniform AC and NC. Constant-depth Boolean circuits. The Yao–Hastad theorem.

Parallel computers, PRAM. Element distinctness in constant time. Computing MAX with n^2processors and also with n processors. Sorting with n processors in $O(\log^2 n)$ time. Scalar product, matrix multiplication in parallel. Computing all-pairs shortest paths in a weighted graph in parallel. The Brent principle; application when fewer processors are available.

References:

1. László Lovász: Complexity of Algorithms, Lecture Notes, Eötvös Loránd University Institute of Mathematics, 2024.
2. C. H. Papadimitriou: Computational Complexity, Addison Wesley, 1994.