Problem set 10: Undirected graphs

Question 1: Draw the following undirected graph: $G = (V, E, \varphi)$, $V = \{A, B, C, D\}$, $E = \{e_1, e_2, e_3, e_4\}$, $\varphi = \{(e_1, \{A, B\}), (e_2, \{B, C\}), (e_3, \{A, C\}), (e_4, \{C, D\})\}$. Determine $d(A)$, $d(B)$, $d(C)$ and $d(D)$. Draw $\overline{G}$, and draw a different graph isomorphic to $\overline{G}$. Are G and $\overline{G}$ isomorphic?

Question 2: Can a 7-vertex simple graph have the following degree sequence?
(a) $4, 4, 3, 3, 2, 2, 1$
(b) $6, 3, 3, 3, 3, 2, 0$
(c) $5, 5, 5, 2, 2, 2, 1$
(d) $2, 2, 2, 2, 2, 2, 2$

Question 3: Does a 9-vertex graph exist with the following degree sequence? And simple graph?
(a) $7, 7, 7, 6, 6, 6, 5, 5, 5$
(b) $6, 6, 5, 4, 4, 3, 2, 2, 1$
(c) $2, 2, 3, 5, 6, 6, 6, 8, 8$

Question 4: Does an 8-vertex graph exist with the following degree sequence? And simple graph?
6,6,6,6,3,3,2,2

Question 5: Prove that any simple finite graph with at least two vertices contains two distinct vertices with the same degree.

Question 6: Prove that if a connected graph with at least two vertices has fewer edges than vertices, then it contains a vertex of degree 1.

Question 7: Prove that every finite graph in which the degrees of all vertices are at least 2 contains a cycle.

Question 8: Let $G = (V, E)$ be a connected graph, and $e \in E$ a bridge. Prove that e cannot belong to any cycle in G.

Question 9: Let $G = (V, E)$ be a simple graph with $|V| = 6$. Prove that either G or $\overline{G}$ contains a subgraph that is a 3-vertex complete graph.