Combinatorial Optimization and Applications

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Reliable solutions using combinatorial optimization

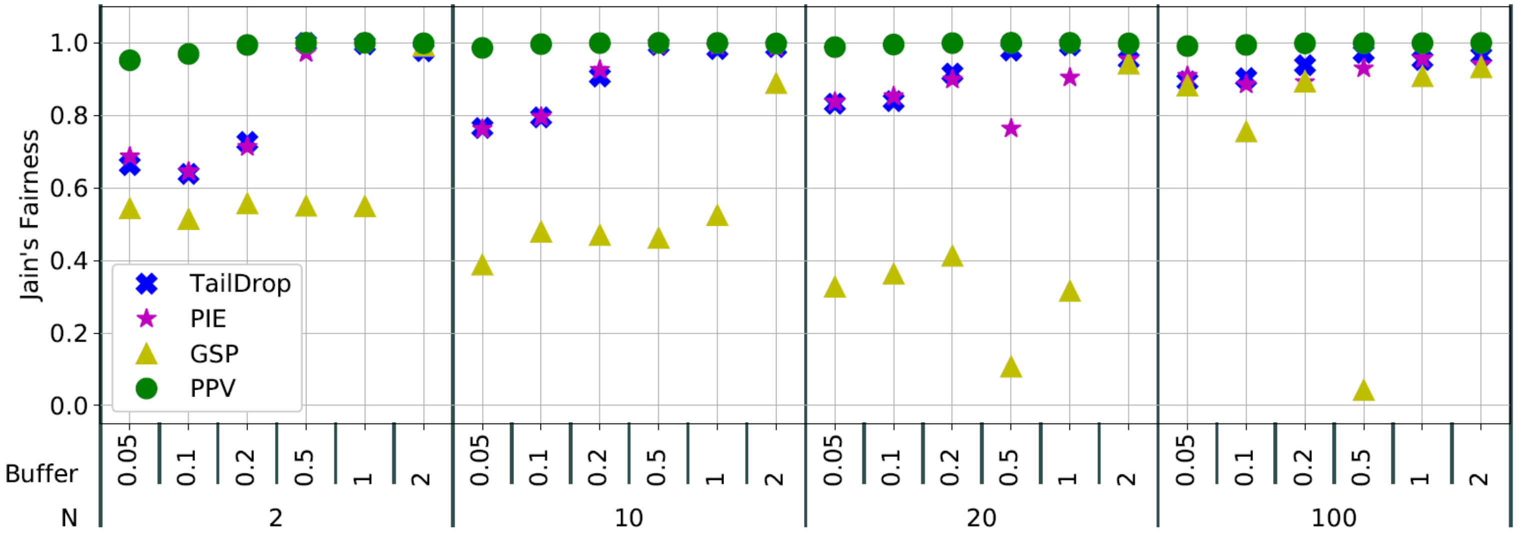
- Provably efficient algorithms
  - Upper bounds on the running time
  - Guaranteed correctness within the mathematical model
  - Guaranteed quality of solutions
- Widely applicable results
  - Can be used in any application that fits the mathematical model
  - Results do not depend on specific implementation issues

Two examples of mathematical contributions to this project:

- Mathematical analysis of the Per Packet Value (PPV) framework
- Algorithms for scheduling problems

The Per Packet Value framework

- PPV is a proposed framework for resource sharing in networks
- Developed by ELTE in cooperation with Ericsson Research
- PPV aims to achieve fairness at a low control cost
- Should accomodate multiple Congestion Control (CC) algorithms



Multi-CC fairness (1Gbps, 10ms)

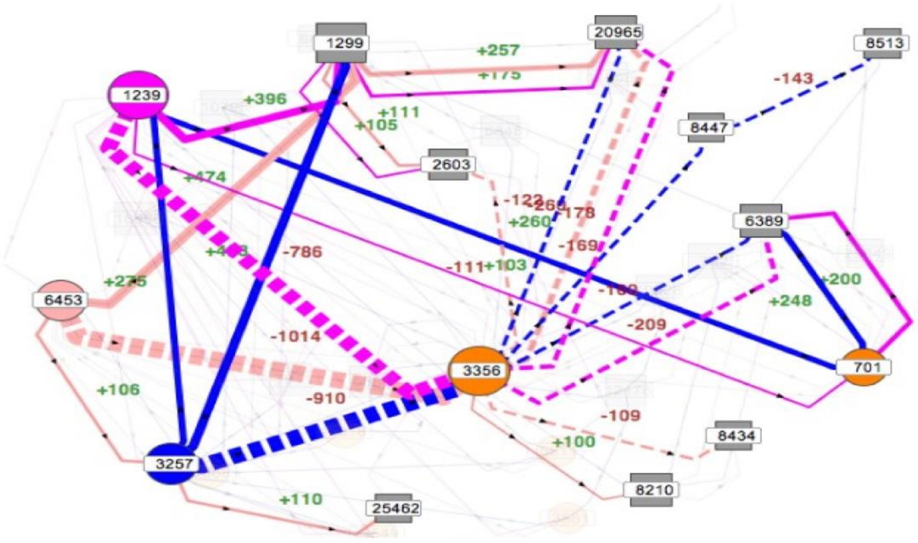
Mathematical analysis of PPV

- We developed a mathematical model of the PPV framework
- We showed that a fair and stable state exists in any network
- We showed that convergence to a fair state is guaranteed for a large class of CC algorithms
- We showed that under PPV, classical CCs achieve sufficient throughput even against aggressive CCs, in any network.

These results confirm that PPV is a viable solution for fair resource sharing in networks with heterogeneous congestion controllers

PPV – future work

- Give bounds on the rate of convergence
- Examine other aspects of fairness
- Analyze the performance of PPV in multi-path routing scenarios



Route changes in a network [Lad, Massey, Zhang: Visualizing Internet Routing Changes, 2006]

Scheduling problems

- Scheduling involves assigning dates to tasks (jobs) under various contraints
- Applications:
  - Manufacturing
  - Timetabling
  - Crew scheduling
  - Financial planning
  - Etc.
- Most scheduling problems are hard: finding the optimal solution is not possible for large instances
- Reliability can be achieved using algorithms that are guaranteed to give near-optimal solutions for any instance

Scheduling under resource constraints

- Resource constraints arise when
  - Each job consumes a fixed amount of a resource
  - Various quantities of the resource arrive at specified dates
  - Jobs cannot be processed until the resource is available
- Examples
  - Availability of raw materials in manufacturing
  - Energy consumption
  - Availability of funding

We have developed general reliable algorithms that can be used for all of the above examples

Our results

- (1+ε)-approximation for arbitrary small ε if processing times are negligible
  - Examples: financial scheduling, production using rare raw materials
- 3/2-approximation if jobs have identical resource requirements
  - Example: ingredient/activity required for every job
- 4-approximation if resource arrival times are unknown

- Joint work with TU Hamburg
- Presented this week at the International Symposium on Combinatorial Optimization